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Curriculum Alignment and Problem-Solving Instruction in Mathematics: Evaluating Azerbaijani Textbooks in Light of Modern Pedagogical Demands

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Abstract: This article examines the extent to which mathematics textbooks align with modern curriculum demands for problem-solving instruction. A review of literature shows that problem solving has become a central objective of mathematics curricula worldwide, framed as both a goal and a method of teaching. Effective problem-solving instruction requires aligning curriculum standards, textbook content, and classroom practices to emphasize higher-order thinking and real-world application. We discuss the concept of curriculum alignment and its importance, drawing on studies of alignment between learning objectives and educational materials. An analysis of mathematics textbooks – including a case example from recent curriculum reforms in Azerbaijan – highlights common misalignments: problem-solving tasks are sometimes superficially included or pitched at inappropriate difficulty levels, and cognitive demands in textbooks may not fully reflect those in curriculum standards. We identify challenges such as inconsistent integration of problem-solving across topics, insufficient guidance for complex real-life problems, and a tendency to focus on rote procedures over strategy development. Recommendations are offered for improving alignment, including designing textbook tasks that foster deep understanding and inquiry, ensuring a progression of problem-solving experiences, and supporting teachers' pedagogical content knowledge. Aligning textbooks with modern problem-solving demands is critical for developing students' analytical skills and for the overall coherence of reform-oriented mathematics education.

Keywords: *Problem-solving; Curriculum alignment; Mathematics textbooks; Cognitive demand; Instructional design; Bloom's taxonomy; Educational reform*

Introduction

Around the world, mathematics education reforms have increasingly emphasized problem-solving proficiency as a key learning outcome. National curriculum standards and international benchmarks (e.g., PISA) now highlight the ability to solve complex, real-world problems as central to mathematical literacy. Research in mathematics education likewise advocates elevating problem solving from a peripheral topic to the core of curriculum and instruction. Hiebert et al. (1996), for example, argued that meaningful reform in math teaching should be based on allowing students to *"problematize"* mathematical content – that is, to engage with mathematics by raising questions, investigating patterns,

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and searching for solutions, rather than simply absorbing facts and practicing routine procedures. This vision positions problem solving not only as an ultimate goal of learning but also as the primary means through which students learn mathematics.

Realizing this vision requires significant shifts in curriculum design and teaching practice. It is not enough for policy documents to call for problem solving; the implemented curriculum must align with these goals at every level, from textbooks and lesson plans to assessments and teacher pedagogy. Misalignment can undermine reform: if textbooks remain focused on rote exercises or if assessments only value recall of procedures, teachers and students receive mixed messages that can dilute the impact of problem-solving initiatives (Martone & Sireci, 2009). Ensuring alignment between intended curriculum standards, the instructional materials (textbooks), and classroom instruction is therefore critical for reform success (Martone & Sireci, 2009). In particular, textbooks play a pivotal role as they often mediate the curriculum for teachers and learners – conveying which topics to teach, which skills to practice, and which problems to solve. Textbooks that are well-aligned with modern problem-solving demands can be powerful enablers of reform; poorly aligned textbooks, by contrast, may impede the development of students' higher-order thinking skills.

This article evaluates how well mathematics textbooks align with contemporary curriculum demands for problem-solving instruction. We begin by reviewing the role of problem solving in modern mathematics curricula and the theoretical frameworks that inform teaching "through" problem solving. We then discuss the concept of curriculum alignment and why it is vital to examine alignment not only in assessments (as is often done) but also in instructional resources such as textbooks. Drawing on recent studies, including content analyses of textbooks and alignment studies based on learning taxonomies, we identify common strengths and gaps in current textbooks' treatment of problem solving. As an illustrative case, we include an analysis of curriculum and textbook changes in Azerbaijan, where a recent curriculum reform attempted to integrate problem-solving-focused content across grades. This case highlights concrete challenges teachers and students face when curriculum objectives and textbook content are not perfectly synchronized. Finally, we offer recommendations for improving curriculum–textbook alignment in support of problem-solving instruction, emphasizing the need for coherent integration of problem-solving tasks, appropriate cognitive demand, and teacher support.

By shedding light on the alignment issue, we aim to contribute to ongoing efforts to improve mathematics instruction. When curriculum standards, textbooks, and teaching practices are all pulling in the same direction – toward developing students' problem-solving abilities – the chances of achieving this educational goal are greatly increased.

1. Problem Solving in Modern Mathematics Curriculum

Problem solving has evolved from a peripheral topic into a central tenet of modern mathematics curricula. Over the past several decades, research and policy have converged on the idea that learning mathematics is not simply about acquiring isolated skills, but about *using* those skills to solve meaningful problems. This shift is evident in major curriculum reforms worldwide. For instance, the Singapore mathematics curriculum framework, widely regarded as successful, famously places *mathematical problem solving* at the heart of its design. Since the 1990s, Singapore's curriculum has explicitly been organized around problem solving as the central focus, supported by five interrelated components (skills, concepts, processes, metacognition, and attitudes) that collectively enable problem-solving ability[2]. This reflects a high degree of alignment in Singapore's educational system: curriculum goals, textbooks, and pedagogy are all oriented towards cultivating problem solvers. The result, as noted by multiple observers, is a coherent curriculum where students regularly engage in challenging problems and develop a deep understanding of mathematical ideas through that engagement.

At the theoretical level, scholars have described different "roles" that problem solving can play in mathematics education. A classic framework by Schroeder & Lester (1989) distinguishes between teaching for problem solving, teaching about problem solving, and teaching through problem solving. In teaching *for* problem solving, traditional content is taught first and then applied to solve problems; in teaching *about* problem solving, general strategies and heuristics are taught explicitly. The most ambitious role is teaching *through* problem solving, where problem-solving activities are the vehicle for learning new mathematical content. Research indicates that this latter approach yields the richest learning. Olivares, Lupiáñez, and Segovia (2021) note that the *"highest degree"* of integration is achieved when curricula adopt a teaching-through-problem-solving approach, as this allows students to develop new knowledge and deeper conceptual understanding while engaging in the problem-solving process. Recent recommendations therefore encourage mathematics programs to organize instruction around complex, open-ended problems – effectively learning mathematics by doing mathematics (Lester & Cai, 2016). Curricula that implement this approach tend to include more open-ended tasks that connect different topics, fostering an integrated understanding of mathematics. For example, Japan's mathematics curriculum, implemented via structured lesson study, exemplifies teaching through problem solving: lessons are built around a single rich problem that students tackle independently and then discuss collectively, guided by the teacher to discover underlying concepts (Isoda, 2015; Fujii, 2018). This method has been shown to promote deeper reasoning and student engagement.

Underpinning the teaching-through-problem-solving approach are certain principles of effective problem-solving instruction. A literature review by Olivares et al. (2021) identified five key principles needed for successfully implementing problem solving in the math curriculum. These are: (1) Understanding – ensuring students build deep conceptual understanding, rather than just procedural know-how; (2) Reasoning – emphasizing logical reasoning and justification of solutions; (3) Autonomy – developing students' independence and perseverance in tackling problems; (4) Collaboration – using peer discussion and group problem-solving to enhance learning; and (5) Affective factors – nurturing positive attitudes, confidence, and curiosity in mathematics. These principles align with broader educational goals of fostering critical thinking and lifelong learning skills. When applied, they transform the classroom environment: students engage in reasoning and sense-making, often working together on non-routine problems, while teachers facilitate rather than simply demonstrate solutions.

It is important to note that simply adding more word problems to a textbook or curriculum does not automatically fulfill these principles. The *nature* of problem-solving tasks and how they are embedded in instruction matters greatly. High-quality problem-solving tasks are those that connect with important mathematical concepts, allow multiple solution strategies, and require students to analyze and reason – not just apply a memorized formula. They also need to be developmentally appropriate and linked in a progression that builds students' skills over time. This is where alignment comes into play: the design of textbooks and lessons must align with the curriculum's problem-solving goals by featuring tasks that truly embody understanding, reasoning, and so on, at the right level of challenge for students. When a curriculum espouses problem solving but provides mainly routine exercises in its materials, a disconnect arises between the intended curriculum (what is described in standards) and the enacted curriculum (what students actually experience).

In summary, modern mathematics curricula demand that problem solving be front-and-center. The literature and exemplary systems show a consensus that teaching mathematics through problem solving – supported by principles of understanding, reasoning, student autonomy, collaboration, and positive attitudes – can lead to improved mathematical proficiency. Achieving this in practice, however, requires careful alignment of curriculum components. In the next section, we delve into what curriculum alignment entails and why it is especially critical for problem-solving instruction.

2. The Importance of Curriculum Alignment

Curriculum alignment refers to the degree of agreement or match between the objectives set forth in curriculum standards, the content and activities in instructional materials (such as textbooks), and the assessments or evaluations of learning. High alignment means that all components of the educational system are working synergistically toward the same goals; low alignment means there are discrepancies (Martone & Sireci, 2009). Alignment is widely seen as essential for effective education because misalignments can lead to inefficient or inequitable learning. If, for example, textbooks and lessons focus on simple recall of facts while exams test complex problem-solving, students will be unprepared for the assessments. Conversely, if a curriculum emphasizes problem-solving skills but textbooks do not provide opportunities to practice those skills, then the curriculum's goals remain unfulfilled in the classroom. Martone and Sireci (2009) stress that strong alignment among curriculum, instruction, and assessment is necessary to ensure validity in education – that is, to ensure that what is taught is indeed what is tested and valued, and vice versa. In the context of mathematics problem solving, alignment is critical so that students consistently encounter the kinds of cognitive challenges the curriculum expects them to master.

There are multiple dimensions to curriculum alignment. One dimension is content alignment, which means the topics and skills in textbooks and tests match those prescribed by the standards. Another is cognitive alignment, which refers to the level of thinking or complexity – for instance, whether materials engage students in mere recall or in higher-order processes like analysis and evaluation. Modern math curricula that call for problem solving implicitly call for higher-order thinking: analyzing novel problems, devising solution strategies, making connections between concepts, and so forth. Therefore, cognitive alignment is particularly important – the tasks in textbooks should mirror the cognitive demands of the curriculum's standards (Webb, 1997). A third aspect can be contextual or performance alignment: the format in which knowledge is used. For example, if standards expect students to apply math in real-world contexts, then textbooks should include real-world problem scenarios, not just abstract exercises.

Recent research has attempted to measure alignment in various ways. One approach, exemplified by Khoy (2025), is to use a framework like *revised Bloom's Taxonomy* to evaluate whether learning objectives and textbook content address a range of cognitive levels. In a comprehensive cross-disciplinary evaluation, Khoy (2025) examined how well textbooks and syllabi aligned with Bloom's taxonomy levels (Remember, Understand, Apply, Analyze, Evaluate, Create). The study found considerable unevenness: many textbooks concentrated heavily on lower-order cognitive skills (memorization and routine procedures) and had relatively few tasks requiring higher-order thinking like analysis or evaluation. This misalignment suggests that despite rhetoric about 21st-century skills, instructional materials may still be lagging in providing opportunities for creative problem solving and critical thinking. Such findings underscore a gap between modern demands – which call for creative problem-solvers – and the reality of many curricula where those skills are underrepresented in day-to-day learning materials. To truly align with modern demands, curriculum designers must ensure that textbooks incorporate tasks at all cognitive levels, especially the higher ones, in appropriate proportions.

Effective curriculum alignment also involves alignment with assessment. If high-stakes assessments (state exams, international tests) emphasize problem-solving and reasoning, this should be reflected in both the standards and the textbooks. Martone and Sireci (2009) describe methodologies to quantify alignment between standards and assessments, often by coding items according to content and cognitive complexity, then comparing distributions. A well-aligned system would show that the majority of test items correspond to what was actually taught and practiced. When alignment studies find discrepancies – for example, if many assessment items test problem-solving skills that were never practiced in class – it highlights the need to adjust either the materials or the assessments. In practice, some education systems respond to poor alignment by revising textbooks or providing teachers with supplementary problem-solving resources. Others adjust assessments to better reflect what is taught. In either case, the goal is to eliminate mismatches that can disadvantage students.

For problem-solving instruction, alignment is especially vital because problem solving is complex to teach and learn. It requires consistent reinforcement across topics and grades. A curriculum might introduce simple problem-solving tasks in early grades and gradually increase complexity – but this works only if textbooks follow a coherent progression. Additionally, problem solving often cuts across traditional topic boundaries (a single rich problem might involve arithmetic, geometry, and data analysis). This means aligned curricula often encourage integration of content strands. Indeed, one hallmark of problem-solving-oriented curricula is the breaking down of silos between algebra, geometry, etc., in favor of interdisciplinary problem contexts. Textbooks need to reflect this integrated approach by including problems that require knowledge from multiple domains, thereby aligning with the curriculum's intent to develop flexible problem-solvers. Conversely, if textbooks remain partitioned (e.g., all geometry problems separated in one chapter with none appearing elsewhere), they may fail to reinforce the interconnected nature of mathematical problem solving that standards promote. Aligning the curriculum thus might involve not just matching individual standards to tasks, but aligning the overall structure and philosophy of the materials with that of the curriculum framework.

In summary, curriculum alignment ensures that what we *intend* students to learn (as per standards) is what they actually *encounter* in their studies, and ultimately what is *evaluated*. For modern mathematics curricula centered on problem solving, alignment means that problem-solving objectives should permeate textbooks, teaching, and tests in a consistent way. The next section examines the current state of mathematics textbooks in this regard, highlighting findings from research on whether textbooks are living up to the challenge of problem-solving instruction.

3. Alignment of Textbooks with Problem-Solving Goals

Textbooks translate curriculum standards into concrete learning experiences; they provide the problems and exercises that students spend much of their time on. Therefore, the alignment of textbooks with problem-solving goals is a focal point for evaluating overall curriculum alignment. A number of studies have scrutinized mathematics textbooks to see if they reflect contemporary emphases on problem solving and effective teaching practices. The evidence suggests a mixed picture: while many newer textbooks include more problem-solving tasks than in the past, there are still significant gaps in how well these tasks align with the depth and breadth of problem-solving skills that modern curricula seek to develop.

One informative study by Alghamdi (2023) analyzed word problem-solving tasks in six widely used third-grade mathematics textbook series in the United States. Alghamdi's content analysis was guided by five research-based teaching practices for effective mathematics instruction: (1) clear goals, (2) reasoning and problem solving, (3) use of visual representations, (4) mathematical discourse and conceptual understanding, and (5) explicit strategy instruction. The results showed notable variation among textbooks in how well these practices were incorporated. Overall, aspects like encouraging student discussion and conceptual understanding were present most frequently, whereas strategy instruction was incorporated the least. In the analysis of 1,457-word problem tasks, many textbooks did include opportunities for students to explain their thinking or use visuals (like drawings or charts) to solve problems – a positive sign of alignment with practices that build understanding. However, the relative scarcity of explicit strategy instruction (i.e. teaching students specific problem-solving heuristics or methods) indicates a weakness. This means that students using those textbooks might get practice solving problems but may not be learning general strategies for tackling novel problems – a key component of problem-solving proficiency. Furthermore, Alghamdi (2023) found statistically significant differences between textbook series in these aspects, implying that some curricula materials align much more strongly with effective problem-solving pedagogy than others. In poorly aligned textbooks, problem-solving tasks might be fewer, more routine, or lacking support for teachers and students to engage deeply. The study concluded that teachers should critically assess their textbooks' problem-solving content and be prepared to supplement activities to ensure alignment with best practices. In other words, where textbooks fall short, a teacher's pedagogical skill and additional resources become crucial to meet the curriculum's problem-solving objectives.

Another aspect of textbook alignment is whether problem-solving tasks increase in sophistication as students progress through grade levels, matching the curriculum's learning progressions. Ideally, a curriculum will introduce simpler problem-solving experiences in early grades (e.g., basic addition/subtraction word problems in familiar contexts) and then gradually present more complex, multi-step, or open-ended problems in later grades. Textbooks should mirror this progression. However, analyses have found that some textbooks do not consistently do so. In some cases, textbooks in middle grades still emphasize routine exercises and computational practice, with only token inclusion of non-routine problems. This can create a disconnect if the curriculum standards for those grades expect students to be developing higher-order problem-solving skills. The issue is not merely the quantity of word problems or applications, but their quality. For instance, a textbook might include many word problems but all following the same template, which students learn to solve by rote, rather than problems that truly require reflection or different approaches. Such tasks might give the appearance of problem-solving practice but do not align with the curriculum's intent to foster creativity and critical thinking.

Cognitive demand analysis is a useful tool here. Educators Stein and Smith (1998) introduced a framework for classifying mathematical tasks by cognitive demand – from memorization and procedures without connections (low demand) up to doing mathematics and reasoning (high demand). When researchers apply this to textbooks, they often find an abundance of low- and moderate-demand tasks and relatively few high-demand ones. This was echoed by Khoy's (2025) findings in terms of Bloom's taxonomy: textbooks tend to overweigh lower-level cognitive skills. The result is that students may not be sufficiently challenged to think mathematically. Maintaining the cognitive demand of tasks in classroom implementation is also a challenge – even if a textbook problem is high-level, a teacher pressed for time might simplify it or turn it into a procedure. Thus, alignment is not only about the textbook content but also about how the textbook is used. Nevertheless, ensuring that textbooks include a substantial proportion of high-demand tasks is a necessary condition for alignment with problem-solving goals. Textbook developers in some countries have begun to integrate special sections or problem-solving investigations in each chapter to address this need. For example, a chapter on algebra might end with an extended real-world problem that ties together the algebraic concepts in an open-ended way. This is a positive step, but effectiveness depends on how those sections are treated (are they optional enrichment that many teachers skip, or an integral part of the learning sequence?). An aligned curriculum would make such problem-solving investigations a core component rather than an add-on.

A specific challenge in textbook alignment is designing problems that connect across topics, reflecting the integrated nature of real-world problems. In many traditional textbooks, content is compartmentalized: students do geometry in one part of the book and never see it applied when doing, say, arithmetic word problems. Modern problem-solving demands often involve using multiple concepts together (for instance, a single problem might require proportional reasoning, geometry, and data interpretation). Some innovative textbooks now include thematic problem sets or projects – for example, a unit project that requires students to apply skills from different chapters to design something or analyze a complex situation. These kinds of tasks signal high alignment with a problem-solving-oriented curriculum, because they encourage students to transfer and synthesize knowledge. However, integrating content in textbooks can conflict with teachers' expectations or the structure of standardized tests, so not all textbooks embrace this fully. In the case where curriculum standards explicitly call for integration (as many do, via practice standards or process standards), textbooks that remain siloed demonstrate a misalignment that could hinder students from developing a holistic problem-solving ability.

Teacher factors also influence how textbook alignment plays out. Even a perfectly aligned textbook is not self-executing; teachers need pedagogical content knowledge to guide students through challenging problems. Tambara (2015) emphasizes that teachers' understanding of both mathematics content and problem-solving pedagogy is crucial for developing learners' problem-solving skills. If teachers lack the strategies to scaffold difficult problems or to facilitate rich mathematical discussions, they may underutilize or skip the problem-solving elements of a textbook. In this sense, alignment has a human dimension: professional development and teacher support materials in textbooks (like teacher's guides) must align with the curriculum's problem-solving ethos as well. Some textbook series include margin notes or supplementary guides that coach teachers on how to prompt student thinking or how to connect a problem to underlying concepts – these are invaluable for maintaining the intent of problem-solving tasks. Without such support, there's a risk that rigorous problems get turned into procedural drills, defeating their purpose. Tambara's (2015) work in South Africa, for instance, found that many teachers needed help unpacking their content knowledge and teaching skills specifically to better teach problem solving. Thus, part of evaluating textbook alignment is looking at whether teacher-facing components of the curriculum (guides, lesson plans) align with the goal of teaching through problem solving, by encouraging inquiry, providing rich problem contexts, and anticipating student difficulties.

In conclusion, current textbooks show progress but also shortcomings in aligning with modern problem-solving demands. There is greater awareness now that textbooks should include problem-solving and reasoning opportunities; however, the depth of alignment varies. Some textbooks integrate high-level problems and effective practices throughout, while others do so sporadically or at a superficial level. The next section will provide a concrete example by examining how one educational system – Azerbaijan – has implemented a new curriculum and the alignment issues observed in its mathematics textbooks. This will illustrate many of the general points with specific instances of alignment and misalignment in textbook content.

4. Case Study: Problem-Solving Alignment in Azerbaijani Textbooks

Azerbaijan provides an interesting case study of curriculum alignment in mathematics, particularly with regard to problem-solving instruction. In the late 2000s and 2010s, Azerbaijan undertook significant curriculum reforms transitioning from traditional content-based teaching to a competence-based approach, much like other international reforms. Mathematics education was redesigned to be taught as a unified subject ("Mathematics") rather than separated into arithmetic/algebra and geometry, and new standards highlighted problem solving, reasoning, and real-life application as key goals. We examine here the example of Nakhchivan (an autonomous region of Azerbaijan) where these new curricula and textbooks have been implemented, shedding light on how well the textbook content aligns with the curriculum's modern demands.

Curriculum Structure: The reformed math curriculum in Azerbaijan is organized around five "content lines" (or strands) that span all grade levels: *(1) Numbers and operations, (2) Algebra and functions,* *(3) Geometry, (4) Measurement, (5) Statistics and probability*. These mirror the broad areas found in many national standards. Importantly, the curriculum specifies not only content topics but also *general competencies and process skills*, such as logical reasoning and problem solving. For each grade, the curriculum outlines general learning outcomes – for example, by the end of Grade 9, students should be able to apply mathematical knowledge to solve various real-life problems, reason logically, and communicate their thinking clearly. These outcomes align with modern demands: they emphasize application, analysis, and interpretation over rote calculation. In principle, then, the intended curriculum strongly values problem-solving ability. The role of textbooks in this system is to realize these outcomes through lessons and exercises. The new textbooks were expected to integrate the five content lines in a coherent way and embed problem-solving tasks throughout.

Integration and Progression: In practice, the new textbooks made significant changes from the old ones. Instead of teaching topics in isolation, the textbooks often interweave different content lines within a single chapter or unit. For example, the Grade 5 mathematics textbook introduces multi-digit numbers and arithmetic operations, but alongside these, it incorporates set theory concepts (like Venn diagrams) when discussing number sets, and includes word problems that require interpretation of data (touching on the statistics strand). The idea was to promote *interdisciplinary connections* and show students that mathematics is not fragmented. While this is laudable and aligns with the curriculum's holistic approach, it posed some alignment challenges. Teachers and students encountered situations where a textbook unit would jump between content strands – e.g., from rounding numbers to union of sets and back to arithmetic operations. The curriculum did allow for integration, but the sequence and pacing in the textbook sometimes felt abrupt. In one instance, after covering comparison and rounding of large numbers, the Grade 5 textbook suddenly introduces the concept of sets and set operations (union, intersection) with a non-trivial problem involving Venn diagrams, and then returns to teaching multi-digit addition and subtraction. This cross-cutting approach can support problem solving (since real problems often involve multiple concepts), but only if carefully aligned with students' readiness. Some teachers reported that such arrangement was challenging for learners, as they had to grasp a new abstract concept (sets) in the middle of practicing arithmetic, potentially without enough grounding. Here, alignment issues arose in terms of cognitive load: the curriculum's intent to integrate content was technically followed, but the textbook may not have provided a smooth scaffolding to make that integration student-friendly. A better alignment might have been achieved by either introducing sets with simpler contexts or waiting until students were more comfortable with the prior arithmetic concepts before blending strands.

Problem Complexity: The new textbooks in Azerbaijan enthusiastically include real-world problem contexts, a clear attempt to align with the curriculum's demand for practical problem-solving. For example, the Grade 5 textbook presents a multi-part problem about national oil production figures and the share contributed by the state oil company. The problem text spans an entire page, providing background on the State Oil Company's role and giving a data table of oil output by year, then asks students to answer questions based on the table. The inclusion of such a real-life context is aligned with the idea of making math relevant and developing students' ability to interpret information. However, this particular problem proved to be *too complex* for most 10–11-year-old students. It introduced new terminology ("budget revenue", "expenditure") and required understanding context that was likely unfamiliar to them. While the curriculum did encourage real-life applications, the alignment faltered in matching the context and cognitive demands to the learners' level. Students struggled to even parse the problem situation, which detracted from the mathematical learning intended (working with large numbers and percentages). As a result, teachers noted that such problems needed to be broken down into smaller sub-problems or heavily guided – essentially the teacher had to adapt the textbook task to align it with students' capabilities. This example underscores that alignment is not just about including problem-solving tasks, but selecting appropriate ones. A more aligned approach might have been to use a simpler context (e.g., a school event or familiar business) to practice the same skills without the extraneous complexity that overwhelms learners. It also highlights an implementation issue: the curriculum's broad goal ("apply math to real-life problems") was interpreted by textbook authors in a way that overshot the target for Grade 5, indicating a misalignment in terms of *difficulty and context familiarity*.

Conversely, some tasks in the new textbooks were too simplistic or repetitive, offering little challenge. For instance, portions of the Grade 5 and 6 textbooks dealing with fractions largely repeated content from Grade 4, such as identifying and comparing simple fractions using visual models, but without extending to deeper problems. The curriculum standards expected that by Grade 6 students move on to operations with fractions and solving fractional word problems, yet textbook exercises often revisited earlier concepts (like equivalent fractions or comparing fractions with the same denominator) with minimal new complexity. This kind of misalignment – tasks not being sufficiently advanced given the standards – can result in wasted instructional time and insufficient development of problem-solving. Teachers observed that some stronger students found these tasks trivial, while weaker students didn't benefit much either because the textbook provided no new strategy or insight beyond what they had already learned. To align better with problem-solving goals, these fraction sections could have included richer problems (for example, a puzzle that involves fractions in a real context, or an open-ended task where students must figure out fractions of a quantity in a story scenario). Instead, the textbooks' heavy use of mechanical drills on fractions meant a missed opportunity to align with the curriculum's emphasis on reasoning and application.

Use of Models and Representations: The curriculum promotes using multiple representations (like number lines, area models, or diagrams) to solve problems – a practice that builds conceptual understanding. The textbooks did incorporate some of this approach. For example, to solve word problems involving fractions in Grade 5 and 6, the teacher's guide suggests using the part-whole bar model (a common Singapore-inspired strategy) to visually represent the relationships. One textbook problem describes: "Gülnar spent 2/5 of her money on fruits and 1/5 of the *remaining* money on meat; if 1 kg of meat cost 9 manat, how much money did she have initially?" This is a classic two-step fraction problem. The curriculum expects students to solve such problems and even to illustrate them with models. In class, teachers found that drawing the bar model (or other diagram) greatly helped students understand the structure of the problem (first dividing the whole into fifths, etc.). The alignment here between the intended skill (using visual representations to solve fraction problems) and the textbook implementation was relatively successful – the problem is non-routine and requires reasoning through fractions, and the suggested use of a model aligns with building problem-solving strategies. However, it was noted that not all students could draw the model on their own; significant guidance was needed. Moreover, the textbook did not explicitly provide a step-by-step explanation or diagram for this problem; it was largely up to the teacher's expertise. This points to a subtle misalignment in instructional support: the curriculum values the use of models, but the student textbook sometimes just gives the problem without any hint to use a model, while expecting that approach. An aligned textbook might include a partially completed bar diagram or a hint like "Try drawing a picture of the situation" to cue students. Without it, the burden fell on teachers, which in classrooms with less experienced teachers could result in students resorting to guess-and-check or other less systematic strategies. In summary, the content was aligned (the problem is a worthy problem-solving task for that grade), but the textbook's pedagogical presentation could be better aligned with the curriculum's recommended methodologies.

Introduction of New Concepts: The Azerbaijani curriculum's embrace of an integrated math course (as opposed to separate algebra/geometry courses) meant that some geometric concepts appear earlier and interwoven with other topics. The Grade 6 textbook, for example, introduces the concept of congruent figures (a geometry concept) right after a unit on decimal arithmetic. The curriculum included congruence under the geometry strand to develop spatial reasoning. The textbook defines congruent figures and gives basic properties (like reflections preserve congruence) and a few exercises (e.g., identifying congruent shapes, understanding simple maps). While this content is valid, an issue arose in how it connected to problem solving. The textbook's treatment was largely theoretical, with definitions and isolated examples. There was little alignment with problem-solving contexts – for instance, no problems where students had to apply congruence to solve a puzzle or a real situation (such as figuring out if two shapes will fit in a space). Additionally, some advanced theoretical notes (quoting mathematician A.N. Kolmogorov on the concept of placing one figure onto another via transformations) were included, which, though interesting, were arguably beyond what sixth graders could fully grasp or what the curriculum standards required. This reflects a case where the depth of treatment was misaligned: instead of focusing on developing problem-solving skills using geometry (like having students do simple constructions or reason about shapes in practical tasks), the textbook veered into formal properties and general notes, which aligns more with an old-style academic approach than the new curriculum's practical orientation. Consequently, students might learn the definition of congruence but not necessarily be able to utilize it in problem-solving scenarios. A more aligned approach might have been to include a hands-on activity or problem (e.g., "Design two different shapes that have the same area – are they congruent or not? Explain.") to engage reasoning. This highlights how alignment is not just about *what* content is present, but *how* that content is framed in terms of problem-solving opportunities.

Assessment Alignment: Within the textbooks, each major topic or chapter often ends with a set of review problems or a "summative" exercise set, meant to prepare students for evaluations. The curriculum's assessment approach encourages a mix of item types, including complex problems. However, teachers in Nakhchivan observed that some of the summative problems in textbooks were exceedingly demanding, arguably beyond what students could reasonably do under exam conditions. For example, a Grade 6 summative assessment problem asked: *"A 40%-salt solution of 20 liters is diluted with water to obtain a 20%-salt solution. How much water should be added?"* This is a typical mixture word problem requiring setting up and solving an equation (or logical reasoning with percentages). While solvable, it involves multiple steps and the understanding of percentage concentration – a concept that students at that stage found challenging. In a timed test setting, many struggled. This raised an alignment concern: if summative tasks in the textbook (which mirror what might appear in exams) are too complex relative to what was practiced in daily lessons, then there is a misalignment in *practice-to-assessment*. Either more practice problems of that type should be included in the regular sections (to build skill), or the summative tasks should be moderated. The presence of a difficult problem per se is not bad – it pushes top students – but if the majority of the class cannot tackle it, it suggests that the curriculum implementation (through the textbook) did not adequately prepare them, even though the curriculum standard might expect it. Teachers resorted to reducing the number of such problems or giving hints, thereby somewhat modifying the intended rigor. Aligning curriculum and textbook in this regard would mean ensuring the difficulty of evaluation tasks is consistent with the tasks used for learning. If high-order tasks are expected in assessment, they should be abundantly present (with support) in the learning phase as well.

Summary of Case Insights: The Azerbaijan example illustrates typical alignment challenges in moving toward a problem-solving curriculum. The curriculum sets ambitious goals: use math in diverse contexts, integrate content areas, develop reasoning. The textbooks made bold attempts to reflect these goals by introducing real-life problems, integrating content lines, and including advanced topics. Some alignments were successful (e.g., multi-step fraction problems with visual models, cross-topic connections), but others overshot or undershot the mark (contexts too complex, insufficient scaffolding, or retention of some rote sections). This case underscores the need for *calibrating* textbook problems to the right level and providing enough instructional support for problem solving. It also highlights the critical role of the teacher: where alignment was weak, teacher intervention was the deciding factor in whether students still achieved the learning objective. For instance, teachers who recognized the intent behind a difficult problem could break it down effectively, whereas less experienced teachers might skip it entirely, leading to a gap between the intended and implemented curriculum. Thus, achieving alignment in problem-solving instruction is a multi-faceted endeavor.

5. Strategies for Improving Alignment for Problem-Solving Instruction

Improving curriculum alignment in mathematics textbooks, particularly to meet modern problem-solving demands, requires coordinated effort from curriculum developers, textbook authors, teachers, and assessment designers. Based on the literature and cases discussed, several strategies emerge that can help ensure what is prescribed in theory is what students experience in practice:

- **Embed Rich Problem Tasks as Core Components:** Textbooks should treat problem-solving activities as an integral part of each topic, not as peripheral extras. This means moving beyond the tradition of having a few challenging problems at the end of a chapter. Instead, problem-solving should be woven through the instructional sequence. For example, when introducing a new concept (like linear equations), the textbook can begin with a problem scenario that naturally leads to that concept (a method often called problem-based learning). By doing so, the textbook aligns with a teaching-through-problem-solving approach from the outset. As Olivares et al. (2021) and others recommend, problem solving should have a prominent and explicit role in every stage of the curriculum enactment. Textbook authors can utilize features such as "Exploratory Tasks" or "Investigations" within each lesson to encourage students to reason and conjecture. In designing these, authors should ensure they reflect the curriculum standards and push students to the required cognitive levels. A practical step is to review each standard and ask: does the textbook provide a problem-based experience that addresses this standard in a meaningful way? If not, adjustments are needed.

- **Ensure a Progression of Difficulty and Cognitive Demand:** Aligning with problem-solving goals means not only including challenging tasks but doing so in a progressive manner. Curriculum designers and textbook writers should outline a clear trajectory for problem-solving skills across grades. Each grade's textbook should build on prior ones. One strategy is to classify problems in the textbook by their cognitive demand (e.g., using Bloom's taxonomy or Stein's task levels) and check that there is an increasing trend. If Grade 5 mostly has "Apply" level problems, by Grade 8 there should be plenty of "Analyze" and some "Evaluate/Create" level tasks. Alignment audits can be performed: for each grade, tally how many tasks fall into each category of thinking. If the distribution is skewed too low or doesn't advance year to year, revisions can be made. Also, within each grade, provide differentiated problem sets (often textbooks label them basic, intermediate, advanced). This allows all students to engage at some level, while ensuring that higher-order tasks are present for those ready and to stretch others. It's crucial that the "advanced" problems are not treated as optional; teachers should be supported and encouraged to use them widely. By normalizing challenge, the materials align with the expectation that struggle and inquiry are part of learning math (and not just fast finishing work for a few students).

- **Contextual Relevance and Clarity:** Problem-solving in modern curricula often involves real-world contexts, but these contexts must be chosen carefully. To align with students' experiences and the curriculum's aims, contexts should be relevant and understandable, yet still requiring students to apply mathematics in new ways. Textbook problems can be improved by pilot-testing them with students or soliciting teacher feedback: Did the context aid engagement or create confusion? For instance, if a curriculum standard expects understanding of linear functions through real-life examples, a textbook might include problems about phone plans or distance-time relationships. It should ensure the context does not include extraneous difficulties (unfamiliar vocabulary or too much data). In cases like the Azerbaijani oil production example, a strategy to improve alignment would be to break a complex context into a series of scaffolded sub-problems that lead students stepwise to the larger problem. Another strategy is to use **multiple contexts for the same math concept**, some familiar (to build confidence) and some novel (to stretch students), which aligns with curriculum goals of transfer. By carefully curating contexts, textbooks can maintain alignment with the curriculum's problem-solving objectives without overshooting in complexity.

- **Explicit Strategy Instruction and Metacognition:** As noted in Alghamdi's study, an area where textbooks often fall short is teaching students *how* to solve problems, not just giving them problems to solve. Modern curricula value metacognitive skills – thinking about one's own problem-solving process – and general strategies like making a table, drawing a diagram, or working backward. Textbooks should include sidebars or short sections that introduce these **problem-solving heuristics** in the flow of lessons. For example, a textbook could have a feature called "Strategy Focus" that occasionally pauses to discuss approaches: "Notice how we solved this problem by first simplifying the situation... This strategy is called *simplifying the problem*. Try it in the next example." By making these strategies explicit, textbooks align better with the curriculum's demand that students become adept problem solvers, not just in one problem but across many. Moreover, encouraging students to reflect on their solution methods (perhaps by including reflective questions like "What other method could you have used?" or "Why did that strategy work here?") addresses the **autonomy and metacognitive** principles highlighted by Olivares et al. (2021). In aligned instruction, students gradually internalize these strategies and can approach new problems with confidence. Textbooks and teacher guides should work in tandem: the student book introduces and reinforces strategies, and the teacher guide provides tips on fostering metacognitive discussion.

- **Teacher Professional Development and Guides:** No matter how well-crafted a textbook is, the teacher remains the mediator of the curriculum. Alignment efforts must extend to supporting teachers in implementing problem-solving instruction. Teacher guides that accompany textbooks should explicitly align with problem-solving pedagogy – for instance, suggesting probing questions to ask during problem-solving sessions, pointing out common student misconceptions and how to address them, and providing alternative methods for solving certain problems. These guides can reference the curriculum's competencies (e.g., "This problem addresses the competency of logical reasoning; encourage students to justify each step"). When teachers understand the intent behind tasks, they are more likely to implement them faithfully rather than, say, short-circuiting a challenging problem by demonstrating the solution. Additionally, training workshops or online professional development modules can be aligned with the new textbooks, focusing on how to use them to develop problem-solving skills. Tambara (2015) found that unpacking teachers' pedagogical content knowledge was key – teachers may know the math content, but need support in pedagogy for problem solving. Aligned curricula ensure that the roll-out of new textbooks comes with training on facilitation of rich tasks, differentiation strategies, and assessment of problem-solving processes. Essentially, alignment is strengthened when teachers become co-designers of the learning experience envisaged by the curriculum, rather than passive users of a textbook. A feedback loop can also be established: teachers can report which problems engaged students deeply or which fell flat, informing future textbook revisions to better align with both curriculum goals and classroom reality.

- **Alignment with Assessment Practices:** Finally, to reinforce alignment, assessment (both formative and summative) should consistently reflect the problem-solving emphases of the curriculum. Textbooks can contribute here by including formative assessment problems and sample test items that mirror high-level demands. For example, at the end of each unit, alongside routine review questions, textbooks might include a "Performance Task" – a multi-step problem-solving exercise that students could even do as a project – which is similar in spirit to what might appear in an exam or as a graded assignment. By doing this, textbooks signal to teachers and students that these complex tasks are not optional puzzles but *essential* parts of learning and assessment. Martone and Sireci's work reminds us that if there's a disconnect between what's taught and what's tested, alignment suffers. Thus, curriculum designers often work with examination boards to ensure new problem-solving standards are reflected in exams. When that happens, textbooks usually follow suit. In contexts where teachers write their own tests, providing exemplar test items in the textbook or teacher's guide (all emphasizing problem-solving and reasoning) can align classroom assessment with curriculum goals. Over time, as students get accustomed to seeing challenging, non-routine problems in both their textbooks and their assessments, a culture shift occurs: problem solving becomes a normal part of mathematics learning, not an out-of-the-ordinary event.

Conclusion

The push for stronger problem-solving instruction in mathematics has reshaped curricula across the globe, but success hinges on careful alignment among standards, textbooks, teaching, and assessment. This article has explored how well mathematics textbooks – a central conduit of the curriculum – meet modern demands for problem solving, and what can be improved. The literature demonstrates a clear consensus that problem solving should be at the heart of math education (Hiebert et al., 1996; Olivares et al., 2021). Countries and regions that have embraced this vision, like Singapore or Japan, show that with coherent alignment, students can become adept problem solvers and perform strongly on international measures. In these cases, curriculum documents, textbooks, and classroom practices all reinforce the development of analytical thinking, perseverance, and application of knowledge.

However, the journey toward such alignment is not without challenges. The case of Azerbaijan's reformed curriculum illustrates that even with the best of intentions, misalignments can occur – whether in the form of overly complex tasks, insufficient scaffolding, or remnants of rote learning that linger in new textbooks. These misalignments can confuse or frustrate learners and teachers, potentially blunting the impact of the reforms. The key lesson is that alignment is a dynamic, continuous process. It requires regular review and refinement. Textbook content should be empirically tested against curriculum goals: Are students actually demonstrating the problem-solving competencies described in the standards? If not, where is the breakdown – in the materials, the teaching, or the assessments? Alignment studies like those by Alghamdi (2023) and Khoy (2025) provide valuable feedback by pinpointing specific gaps (for instance, lack of strategy instruction or uneven cognitive level coverage). Curriculum developers and authors can use such findings to make targeted improvements.

It is also evident that teachers are the linchpin in making alignment a reality. As Tambara (2015) emphasizes, teachers need both content knowledge and pedagogical skill to cultivate problem solving. No textbook, no matter how aligned, can single-handedly produce problem-solving proficiency without teachers who understand and embrace the underlying philosophy. Thus, investing in teacher education and aligning it with curriculum goals is as important as aligning textbooks. In fact, one could argue that the ultimate measure of alignment is when teachers, textbooks, and assessments all speak the same language of instruction. When a student moves from classwork to homework to test, they should encounter a consistent expectation to think, to reason, and to problem-solve. Creating this consistency builds student confidence and competence in tackling unfamiliar problems – a key aim of modern math education.

In conclusion, evaluating and enhancing curriculum alignment in mathematics textbooks is crucial for fulfilling the promise of problem-solving-centered reforms. Progress has been made, as seen in the increasing presence of rich tasks and inquiry-based learning in many textbooks. Yet, continuous vigilance and adaptation are needed. Curriculum developers should collaborate closely with textbook authors, teachers, and researchers to ensure that every problem in a book serves a purpose aligned with learning objectives, and that no important skill (like a problem-solving strategy or a cognitive process) is left unsupported. By heeding research insights and on-the-ground feedback, future textbooks can better embody the curriculum's vision. The payoff for achieving high alignment is substantial: students who not only master mathematical procedures but can also **think mathematically** – approaching novel challenges with confidence, creativity, and rigor. These are the problem solvers and critical thinkers that education in the 21st century aims to develop. Aligning our textbooks with these goals is an investment in our learners' success, in mathematics and beyond.

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